HEAT EXCHANGE IN THE NEIGHBORHOOD OF THE STAGNATION POINT ON A PERMEABLE SURFACE

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The problem of dissociated and partially ionized gas flow around the frontal point of a body is considered. Various gases are supplied (injected) through the porous body surface into the laminar boundary layer. Correlation equations are obtained for the heat exchange parameter.

A majority of authors [1-3] considered the boundary layer as a homogeneous gas or as a binary mixture in a theoretical investigation of the influence of an injected mass supply on the heat exchange and friction. However, at temperatures of $\sim 6 \cdot 10^{3}$°K and above, when gas dissociation starts, and then ionization as well, the laminar boundary layer becomes a multicomponent layer. In this case the problem is complicated considerably. The most detailed investigations of the heat and mass exchange in a multicomponent mixture have been carried out in [4, 5]. On the basis of these investigations, the authors proposed a sufficiently simple engineering formula to compute the heat exchange:

$$\frac{(\alpha/c_p)}{(\alpha/c_p)_0} = 1 - 0{,}65 \left(\frac{M_e}{M_v}\right)^{0,4} \bar{G}. \tag{1}$$

Other simple relationships obtained in [6-8], which are in perfectly satisfactory agreement with (1), can be used in addition to compute the heat and mass exchange in the neighborhood of the forward stagnation point. However, all these relations are valid for small values of the injected gas consumption, when $\bar{G}$ does not exceed 1.5.

This paper is devoted to an investigation of the heat exchange on a permeable surface around which a partially ionized gas flows in a broad range of variation of $\bar{G}$.

The problem of a multicomponent laminar boundary layer in which diverse physicochemical processes proceed, is solved in the flow of a partially ionized gas around a permeable surface with coolant delivered through it. Under such conditions the heat exchange on an impermeable surface has already been examined earlier [9], and criterial dependences have been obtained for the heat exchange parameter in the absence of injection.

This investigation is a continuation of [9]. The influence of gas blowing (air–air, and nitrogen–nitrogen mixtures are considered in particular) on the magnitude of the convective heat flux for high free-stream enthalpies and high values of the convective heat flux is studied. The influence of injection on the radiant flux is not considered herein.

The stationary flow of an equilibrium multicomponent gas mixture is described by the following system of equations [9] using the effective gas parameters:

1) continuity equation for the mixture

$$\frac{\partial}{\partial x}(\rho u r) + \frac{\partial}{\partial y}(\rho v r) = 0; \tag{2}$$

2) momentum equation

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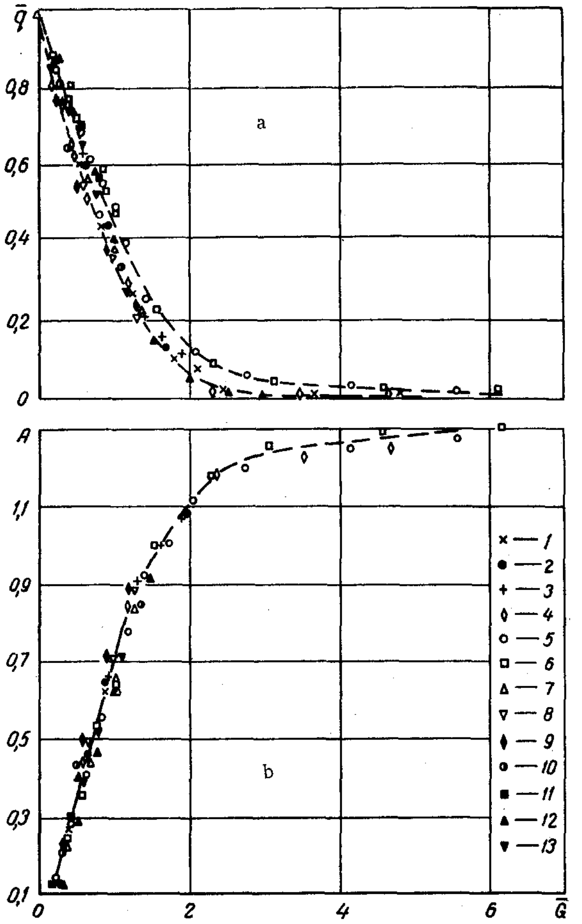

Fig. 1. Dependence of the dimensionless heat flux $\bar{q} = (\alpha/c_p)/(\alpha/c_p)_0$ (a) and the parameter $A = [1-(\alpha/c_p)/(\alpha/c_p)_0]/(M_e/M_v)^{0.25}$ (b) on the dimensionless coolant consumption. Air: 1) $T_e = 6\cdot10^3$°K; 2) $8\cdot10^3$; 3) 10^4; 4) $12\cdot10^3$; 5) $14\cdot10^3$; 6) $15\cdot10^3$; nitrogen: 7) $T_e = 6\cdot10^3$°K; 8) $8\cdot10^3$; 9) 10^4; 10) $12\cdot10^3$; 11) $14\cdot10^3$; 12) data of N. A. Anfimov [4]; 13) data of V. P. Mugalev [6].

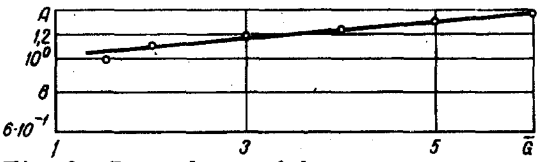

Fig. 2. Dependence of the parameter $A = [1-(\alpha/c_p)/(\alpha/c_p)_0/(M_e/M_v)^{0.25}$ on $\bar{G}$ for large injections.

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{dp}{dx}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right); \tag{3}$$

3) energy equation

$$\rho u\frac{\partial I}{\partial x}+\rho v\frac{\partial I}{\partial y}=\frac{\partial}{\partial y}\left(\frac{\lambda_{\text{eff}}}{c_{p\,\text{eff}}}\frac{\partial I}{\partial y}\right)+\frac{\partial}{\partial y}\left[\mu\left(1-\frac{1}{\text{Pr}_{\text{eff}}}\right)\frac{\partial}{\partial y}\left(\frac{u^2}{2}\right)\right]. \tag{4}$$

The boundary conditions are the following:

$$y=0\quad u=0;\quad v=0;\quad I=I_w=\text{const};\quad (\rho v)_w=\text{const};$$
$$y\to\infty\quad u\to u_e;\quad v=0;\quad I\to I_e.$$

The coordinate x is here directed along the body surface, and the coordinate y along its normal.

Using the transformation

$$\xi=\int_0^x \mu_w\rho_w u_e r^2 dx,\qquad \eta=\frac{\rho_e u_e r}{\sqrt{2\xi}}\int_0^y\frac{\rho}{\rho_e}dy, \tag{5}$$

we reduce the system (2)-(4) to a system of ordinary differential equations.

The system (2)-(4) in the new independent variables (5) now is:

1) momentum equation

$$(lf'')'+ff''+\frac{1}{2}\left[\left(\frac{\rho_e}{\rho}\right)-(f')^2\right]=0. \tag{6}$$

2) energy equation

$$\left(\frac{l}{\text{Pr}_{\text{eff}}}g'\right)'+fg'=0. \tag{7}$$

The boundary value problem is solved with the boundary conditions

$$\eta=0\quad f_w=\text{const};\quad f'_w=0;\quad g=g_w;$$
$$\eta\to\infty\quad f'(\infty)\to 1;\quad g(\infty)\to 1. \tag{8}$$

The system of equations (6), (7) with the boundary conditions (8) was solved by a numerical method.

Computations of the heat exchange from the hot gas to the wall were performed for two cases.

In the first case, the heat exchange was examined in the neighborhood of the forward stagnation point of the permeable body when a stream of partially ionized air flowed around this body. The thermodynamic and transport properties of air were taken from [10, 11]. The computations were carried out in a broad range of variation of the stagnation enthalpy ($I_0 = (12.75\text{-}115)\cdot10^3$ (kJ/kg)); both the dissociation domain (from $T_0 = 6\cdot10^3$°K), so that the computed results on the heat exchange could be compared with the results of other authors [4-6], and the ionization domain (to $T_0 = 15\cdot10^3$°K) were hence considered. Air with the same thermodynamic and transport properties as in the free stream was used as coolant. This assured a more correct solution of the problem with injection in taking account of a multicomponent medium. In investigating the heat exchange, the problem of high-temperature air flow around the frontal point of an impermeable body was solved first in order to determine the generalized coefficient of heat exchange $(\alpha/c_p)_0$ in the absence of injection. The surface temperature was given as the boundary condition on the wall ($T_W = 1000°K$), and therefore, so was the gas enthalpy at the wall temperature (g_W). Determination of the generalized heat-exchange coefficient in the presence of injection (α/c_p) was the next step in the solution. The surface temperature hence remained at $T_W = 1000°K$. The heat-exchange computations on a permeable surface were performed in a broad range of variation of the injection parameter ($-f_W = 0$–2.0) since the coolant consumption is defined as

$$(\rho v)_w = -f_w \sqrt{2\mu_w \rho_w \left(\frac{du_e}{dx}\right)_0}. \tag{9}$$

Analogous computations were carried out for the case of high-temperature nitrogen, whose thermodynamic and transport properties were taken from [12, 13], flowing around the frontal point of a permeable body. In this case, nitrogen was used as the coolant injected through the porous surface into the boundary layer.

The results of computing the heat exchange for air and nitrogen in the presence of injection are presented in Fig. 1a. Presented in this figure is the dependence of the dimensionless heat flux $\bar{q}$, or $(\alpha/c_p)/(\alpha/c_p)_0$, on the dimensionless injection parameter $\bar{G} = (\rho v)_W/(\alpha/c_p)_0$. It is seen that the results for air and for nitrogen lie on a single curve. This is explained by the fact that the thermodynamic and transport properties of these gases, as well as the molecular weights, do not differ radically. Moreover, it can be noted that the higher the free-stream enthalpy (and therefore, the convective heat flux as well), the greater should the coolant consumption be to protect the wall from the hot gas. As is seen from Fig. 1a, the curves of the dependence of the dimensionless heat flux $\bar{q} = (\alpha/c_p)/(\alpha/c_p)_0$ on $\bar{G}$ are stratified for different free-stream temperatures (or enthalpies). This is explained by the change in molecular weight with temperature. If a correction for the molecular weight is introduced in the form of the ratio $(M_e/M_v)^{0.25}$, then all the computed points for all free-stream temperatures lie well on the single curve (Fig. 1b). The results of Mugalev [6] and of Anfimov and Al'tov [4], which lie well on this curve, are superposed here. As is seen from Fig. 1b, the dependence of the dimensionless heat flux $\bar{q}$ on the dimensionless injection parameter $\bar{G}$ is linear in the domain of small injections (to $\bar{G} < 1.5$), and can be represented for $\bar{G} < 1.5$ as the following dependence:

$$\frac{(\alpha/c_p)}{(\alpha/c_p)_0} = 1 - 0.67\left(\frac{M_e}{M_v}\right)^{0.25}\bar{G}. \tag{10}$$

In the domain of high coolant consumptions, the dependence $(\alpha/c_p)/(\alpha/c_p)_0 = f(\bar{G})$ (Fig. 2) can be represented for $\bar{G} \geq 1.5$ as

$$\frac{(\alpha/c_p)}{(\alpha/c_p)_0} = 1 - \left(\frac{M_e}{M_v}\right)^{0.25}\exp\left[2.303\cdot 10^{-1}(-0.45 + 0.3\bar{G})\right]. \tag{11}$$

In the case of injection of a mixture of gases through a permeable surface, the dependences (4.1) and (4.2) obtained by Anfimov in [5] must be used.

NOTATION

x, y	are the coordinates;
u, v	are the velocity components, m/sec;
ρ	is the density, kg/m^3;
p	is the pressure, bar;
T	is the temperature, °K;
μ	is the mixture viscosity, kg/m · sec;
M	is the molecular weight;
I_0	is the total mixture enthalpy, kJ/kg;
λ_{eff}	is the effective heat conduction coefficient, kW/m · °K;
$c_{p\,eff} = (\partial I/\partial T)_p$	is the effective specific heat, kJ/kg · °K;
$Pr_{eff} = \mu \cdot c_{p\,eff}/\lambda_{eff}$	is the effective Prandtl number;
l	is the compressibility parameter;
f	is the stream function;

$f' = u/u_e$	is the dimensionless velocity;
$(\alpha/c_p)_0 = q_{w,0}/(I_0 - I_w)$	is the generalized coefficient of heat exchange in the absence of injection, $kg/m^2 \cdot sec$;
$(\alpha/c_p) = q_w/(I_0 - I_w)$	is the generalized heat exchange coefficient in the presence of coolant injection, $kg/m^2 \cdot sec$;
$q_{w,0}$	is the convective heat flux on an impermeable surface, kW/m^2;
q_w	is the convective heat flux on a permeable surface, kW/m^2;
$\bar{q} = q_w/q_{w,0} = (\alpha/c_p)/(\alpha/c_p)_0$	is the dimensionless heat flux at the wall;
$\bar{G} = (\rho v)_w/(\alpha/c_p)_0$	is the dimensionless coolant consumption.

Subscripts

w	denotes the wall;
e	denotes the outer boundary;
0	denotes the stagnation parameters;
eff	denotes the effective total;
v	denotes the injected gas;
'	denotes the differentiation with respect to the coordinate η.

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